

Undergraduate

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REQUEST FOR  
**ADDITION OF COURSE TO  
CORE CURRICULUM**

College/School: Arts & Sciences  
Department: Mathematics

Subject Prefix: MATH Course Number: 1190 Semester Credit Hours: 3

TCCNS Number (if applicable) MATH Hours Per Week: 3 Lecture  
1325  
(common course number)

Title Business Calculus \_\_\_\_\_ Lab  
Short Course Title : \_\_\_\_\_ Recitation  
(maximum 22 characters including spaces) \_\_\_\_\_ Other

Category of Core Curriculum course is to be added: Mathematics

Catalog Description:

Differential and integral calculus with emphasis on applications to business.

Prerequisite(s):

Two years of high school algebra and consent of department; or MATH 1100 with grade of C or better.

If course is cross-listed, indicate below:

Department: \_\_\_\_\_ Subject Prefix/Course Number: \_\_\_\_\_  
Department: \_\_\_\_\_ Subject Prefix/Course Number: \_\_\_\_\_

Justification for course to be added to Core Curriculum (Include how course would satisfy each exemplary objective.):

**Exemplary Educational Objectives**

*1. To apply arithmetic, algebraic, geometric, higher-order thinking, and statistical methods to modeling and solving real-world situations.*

The "higher-order" concept of the limits are used to model instantaneous rates of change, such as instantaneous rates of production, common in the study of business and economics. Limits are looked at from arithmetic, algebraic, and geometric viewpoints. Rates are visualized geometrically as slopes of tangent lines. Profit, revenue, and cost are modelled geometrically as the area under a marginal rate curve. The concept of probability density makes the connection between calculus and statistical methods. The unit on applied optimization problems allows students to use a variety of algebraic functions to model real-world situations, and optimal solutions are found by either graphical or algebraic means.

*2. To represent and evaluate basic mathematical information verbally, numerically, graphically, and symbolically.*

The course concentrates on the fundamental ideas of limits, derivatives, and integrals. Derivatives are explored numerically by looking at average rates of change over small time intervals, they are discussed verbally in the context of marginal rates and associated applications, they are explored graphically as the slope of a tangent line, and students learn a number of symbolic techniques, such as the Chain Rule, for calculating derivatives. Integrals are explored numerically, as Riemann sums, they are explored verbally in the context of computing values of continuous income streams, they are visualized graphically as areas under curves, and students learn various symbolic techniques, such as substitution, to work with integrals.

*3. To use appropriate technology to enhance mathematical thinking and understanding and to solve mathematical problems and judge the reasonableness of the results.*

Graphing calculators are integrated into the course. The calculators are used as a graphing tool so students can get a good intuitive feel for how the graphs of different types of functions look and how changing parameters affects the shape of the graph. The “zoom” feature of the calculator helps students to visualize the fundamental concepts of limit and derivative. The tabular features on the calculator help the students look at numerical results in a less tedious fashion, and let them do numerical exploration of limits, derivatives, and integrals. Students work with examples where using a tangent line approximation gives reasonable results over the short-term, but gives unreasonable results further out.

*4. To interpret mathematical models such as formulas, graphs, tables and schematics, and draw inferences from them.*

Students learn the practical meaning of derivatives and associated mathematical notation in the real-world context of marginal cost, marginal revenue, marginal profit, *etc.* They learn to make inferences about optimal solutions to problems in business by making a careful analysis of a graph.

### **Over-arching Objectives**

*1. explore math*

During the course, students explore two instances, the derivative and the integral, of a profound and fundamental concept in modern mathematics, the limit. Derivatives and integrals are explored from many different directions and in numerous different contexts.

*2. make connections between different areas of knowledge and different ways of knowing*

Students learn connections between abstract mathematical theory and its implications in business and economics. They experience “knowing” a function from different perspectives: symbolic (or algebraic), numerically, graphically (or geometrically), and verbally.

*3. be able to locate, evaluate and organize information including the use of information technologies*

To develop a mathematical model, students need to extract essential information from the description of the problem or the given data. That information must then be organized into an abstract mathematical model or formula, which can then be evaluated, often with the assistance of a graphing calculator.

*4. think critically and creatively, learning to apply different systems of analysis*

Mathematical problems are analyzed symbolically, graphically, numerically, and verbally. The more challenging problems in the course require critical and creative thinking to re-arrange the data given into a more familiar form so that one of the standard methods taught in the course may be applied more directly. The conceptual exploration of limits and infinite sums requires critical and creative thought and can lead to discussions of some “paradoxes.”

*5. develop problem solving skills*

Problem solving skills are developed throughout the course. Students are taught to systematically identify information given to them and to organize it into a coherent mathematical model. They are asked to identify the assumptions underlying the problem. Students get practice in assigning mathematical variables in such a way as to use fundamental ideas in calculus to create tractable algebra problems. After employing algebra to find a mathematical solution, they interpret the meaning of that solution to the original situation.

*6. cultivate self-responsibility, building a foundation for life-long learning*

Throughout the course, students reinforce their familiarity with technical mathematical terminology and symbolism. They also develop technical proficiency with symbolic calculation rules for derivatives and integrals, techniques they will need throughout their business and economics courses and later in life for quantitative decision making. Students are responsible for reading the textbook and learning some of the material in the course on their own, they are responsible for completing their assignments in a timely manner, and they are responsible for seeking out help and assistance outside of class. All of these activities will leave them well-prepared to continue learning mathematics on their own outside of a formal mathematics class, and to apply mathematics during their other courses while still a student and in their careers later in life.

Consultation with University Curriculum Assessment Committee member:

Revised VPAA: 11/00  
UCC-A-102

Department: Mathematics Contact: William Cherry Date: 04/19/2009

New Core Curriculum Requests must include:

- Syllabus:  Maximum 4-page syllabus attached  
Assessment:  Consultation w/University Curriculum Assessment Committee member in this core component group.  
 Assessment procedures (criteria to be used in assessing this course) must be attached separately

**APPROVED:**

Department Chair: J. Matthew Dyer Date: 4/24/09  
College/School Curriculum Committee Chair: B. Siluaha Date: 5/7/09  
Dean of College/School: B. Siluaha Date: 5/7/09  
Core Oversight Committee Chair: \_\_\_\_\_ Date: \_\_\_\_\_  
University Curriculum Committee (VPAA): \_\_\_\_\_ Date: \_\_\_\_\_

# **Math 1190 (Business Calculus) Core Assessment Plan**

## **Mathematics Department Core Assessment Plan**

The Mathematics Department will assess core courses using a combination of questions on final exams targeted toward individual exemplary educational objectives and over-arching objectives and examination of project portfolios. The Department's Undergraduate Affairs Committee will review the assessment data for each of the department's core courses on a rotating basis. The Department's courses will be grouped into four groups, according to target audience: Group I will include courses intended for general university students: Math 1580/1581 and Math 1680/1681; Group II will include courses intended primarily for business students: Math 1190 and Math 1400; Group III will include courses intended primarily for math, science, and engineering students: Math 1600, Math 1610, Math 1650, Math 1710; and Group IV will include courses intended for elementary education majors: Math 1350 and Math 1351. The Undergraduate Affairs committee will review the assessment data for one group of courses each academic year and make recommendations about how the courses and the assessment process can be improved.

## **Targeted Questions on Final Exams**

The exemplary educational objectives and most of the over-arching objectives will be assessed by targeted questions on the final exam. The Mathematics department will create a database of final exam questions that specifically target the objectives (examples given below) and all instructors will be required to include a certain number of questions on their final exams from this question database. Instructors will record and report the number of students getting these individual questions correct. The department will keep these statistics for review by the department's Undergraduate Affairs Committee. Not all objectives will be assessed in every section during every semester, but a rotation schedule will be set-up so that every objective is assessed periodically.

## Examples of possible final exam questions targeting Exemplary Educational Objectives

1. *To apply arithmetic, algebraic, geometric, higher-order thinking, and statistical methods to modeling and solving real-world situations.*

Chris accepts a job as a truck driver at the age of 25. Assume retirement at age 65, an annual salary of \$41,000 that is paid as a continuous money flow, and a prevailing interest rate of 7% compounded annually.

- What is the accumulated present value of the money flow?
- Describe how to represent the accumulated present value of the money flow as an area.

Or

The “time to failure”  $t$ , in hours, of a machine is often exponentially distributed with probability density function  $f(t) = \frac{e^{-t/a}}{a}$ , where  $a$  is the average amount of time that will pass before a failure occurs.

- If  $a=100$  hours, what is the probability that a failure will occur in 50 hours or less?
- Describe how to represent your answer in (a) as an area.

2. *To represent and evaluate basic mathematical information verbally, numerically, graphically, and symbolically.*

Consider the following graph of Demand  $D(x)$  as a function of price  $x$ .

[Graph omitted here]

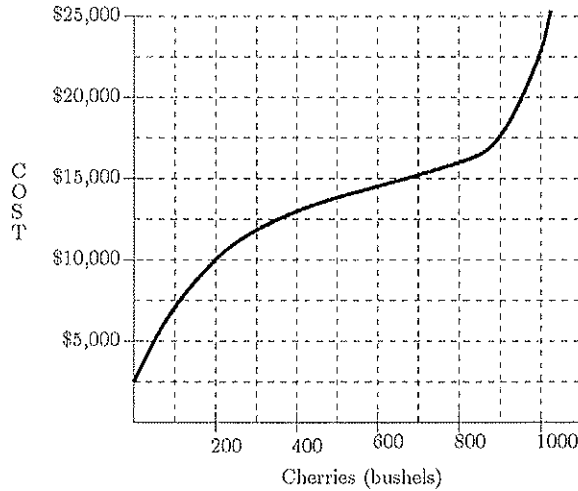
- Using the graph above, illustrate how you can visualize the “average Demand”  $D(x)/x$  as a slope.
- Using the graph above, illustrate how you can visualize the marginal demand  $D'(x)$  as a slope.
- At the point  $x=20$ , which slope is steeper, the slope representing average demand or the slope representing marginal demand? Does this tell you you are in the range of inelastic or elastic demand?
- If  $D(x)=100e^{-0.1x}$ , calculate the elasticity of demand when  $x=20$ .

3. *To use appropriate technology to enhance mathematical thinking and understanding and to solve mathematical problems and judge the reasonableness of the results.*

- With the assistance of a graphing calculator, sketch the graph of  $f(t) = t^2e^{-t}$ .
- With the assistance of a graphing calculator, compute  $\int_0^{50} f(t)dt$ .
- Comment on the reasonableness of using  $f(t)$  as a probability density on the time interval  $[0, \infty)$ . If  $f(t)$  is not suitable as a probability distribution, could some simple modification of  $f(t)$  be a reasonable probability distribution?

4. To interpret mathematical models such as formulas, graphs, tables and schematics, and draw inferences from them.

(10 points) William owns a cherry farm, and the following graph shows how much it costs William to grow cherries, as a function of the number of cherries he grows.



- (A) (2 points) How much does it cost William to grow 200 bushels of cherries? \_\_\_\_\_
- (B) (2 points) What is William's *average cost* if he produces 200 bushels of cherries? \_\_\_\_\_
- (C) (2 points) On the graph above, sketch a line segment whose *slope* represents William's average cost of producing 200 bushels of cherries. Label this line segment "C" as in part (C).
- (D) (4 points) Suppose William wants to grow the amount of cherries that minimizes his average cost. Use the graph above to estimate both the number of cherries William should plant and his minimum average cost. No explanation necessary.

Number of bushels William should plant \_\_\_\_\_

William's minimum average cost \_\_\_\_\_

### Examples of possible final exam questions targeting Over-Arching Objectives

2. make connections between different areas of knowledge and different ways of knowing

The total number of cellular phones  $T(x)$ , in thousands, in use in Israel is well-approximated by the formula  $T(x) = \frac{5200}{1+16.3e^{-0.7x}}$ , where  $x$  denotes the number of years since 1994 and is represented by the graph.

[Graph omitted here]

- a) Estimate the total number of cellular phones that were in use in Israel in 2005 using the formula and explain how you can visualize this quantity on the graph.

- b) Using algebraic limit rules, find  $\lim_{x \rightarrow \infty} T(x)$  and explain how to visualize this limit on the graph.
- c) Using symbolic differentiation rules, compute  $T'(11)$  and explain how to visualize this quantity on the graph.

*4. think critically and creatively, learning to apply different systems of analysis*

The value of good wine increases with age. Thus, if you are a wine dealer, you have the problem of deciding whether to sell your wine now, at a price of \$100 per bottle, or to sell it later at a higher price. On the other hand, if you wait to sell it later at the higher price, you face an opportunity cost of not being able to invest the money you would get by selling the wine now. Suppose you estimate that the price a wine connoisseur would be willing to pay for this wine  $t$  years in the future is  $100(1 + 20\sqrt{t})$ . What is the optimal time to sell the wine, assuming a prevailing interest rate of 5%, compounded continuously?

*5. develop problem solving skills*

A 24 inch piece of wire is cut into two pieces. One piece is used to form a circle and the other to form a square. How should the wire be cut so the sum of the areas enclosed by the two pieces of wire is minimal? maximal?

**Portfolio of Students Projects to assess the following three over-arching objectives:**

- 1. *explore math*
- 3. *be able to locate, evaluate and organize information including the use of information technologies*
- 6. *cultivate self-responsibility, building a foundation for life-long learning*

The department will maintain a library of projects suitable for group work. All instructors will assign one project as part of their course, although they will have flexibility in deciding what portion of the students' grades will be based on the project and whether and to what extent the project will involve group work. To complete this assignment, students will have to work on a problem that requires them to read, think, and learn about mathematics not directly discussed in class and to use technology such as Microsoft Excel in a more intensive form than typically done in class. Successful completion of the project will demonstrate that students are able to learn and explore mathematics on their own or together with a group of their peers, thus forming a foundation for them to continue learning mathematics after they leave the university. An example of a project that could go into the project library could be using regression to apply a logistic model to analyze motion picture box office revenue. Such a project also allows students to explore the mathematical concept of limit and see how limits relate to the concrete context of the motion picture business. Portions of such a project could be used to assess the above three objectives as follows:

1. *explore math*

Students will create scatter plots of motion picture box office revenue and examine how well they fit various mathematical models: linear, exponential, logistic

3. *be able to locate, evaluate and organize information including the use of information technologies*

Students will be asked to gather revenue data from the internet (*e.g.*, [boxoffice.com/numbers](http://boxoffice.com/numbers)) for recent releases. They will use technology such as a graphing calculator or Microsoft Excel to test various regression models.

6. *cultivate self-responsibility, building a foundation for life-long learning*

Students will need to complete the project outside of class, and be responsible for gathering their own data and learning the appropriate technology.

The department will collect and retain a random sample of the projects completed in Math 1190 each semester. Every fourth year the Undergraduate Affairs Committee will review these portfolios and assess how well Math 1190 is cultivating self-responsibility and building a foundation for life-long learning in the students and giving them opportunities to explore mathematics.

# MATH 1190

## Semester Year/Dates

<b>COURSE/Section #</b> MATH 1190	<b>COURSE TITLE:</b> Business Calculus
<b>INSTRUCTOR:</b>	<b>OFFICE:</b>
<b>OFFICE HOURS:</b> Four (4) hours/wk and also by appt.	<b>OFFICE PHONE:</b>
<b>EMAIL:</b>	<b>CLASS MEETS:</b> Three (3) hours/wk.
<b>WEB ACCESS:</b>	
<b>COURSE DESCRIPTION:</b> 3 hours. Differential and integral calculus with emphasis on applications to business. Satisfies the Mathematics requirement of the University Core Curriculum.	
<b>Prerequisite:</b> Two years of high school algebra and consent of department; or MATH 1100 with grade of C or better.	
<b>TEXT:</b> <i>Calculus and its Applications</i> by Bittinger and Ellenbogen, 9 <sup>th</sup> edition, Pearson 2008	
<b>GRAPHING CALCULATOR:</b> TI 83, TI 83Plus, TI 84 or equivalent.	
<b>MATH LAB:</b> Web site: <a href="http://www.math.unt.edu/mathlab">www.math.unt.edu/mathlab</a> The UNT Math Lab is located in GAB 440 Monday - Thursday: 7 am – 9 pm Friday: 7 am – 4 pm Saturday: Noon – 5 pm (Closed Sundays and holidays)	<b>ATTENDANCE POLICY:</b> Class attendance is mandatory. Students are responsible for all information given in class, regardless of his/her attendance.
<b>MAKE-UP TEST POLICY:</b> Tests and exams must be taken in class as scheduled. Makeup exams will only be given in very exceptional circumstances, such as serious illness, and must be arranged in advance.	
<b>ACADEMIC DISHONESTY:</b> Cheating on final exams, on in-class tests, or on quizzes is a serious breach of academic standards and will be punished severely and generally result in a student failing the course. All work done on in-class exams and quizzes must represent only the student's own work, unless otherwise stated in the directions. See <a href="http://vpaa.unt.edu/academic-integrity.htm">http://vpaa.unt.edu/academic-integrity.htm</a> for details on academic integrity at UNT.	
<b>EVALUATION:</b> Average of in-class exams      60% Homework                              15% Final exam                              25%	<b>GRADE ASSIGNMENT:</b> A: [90%, 100%]; B: [80%, 90%]; C: [70%, 80%]; D: [60%, 70%]; F: [0%, 60%), 59% is an F The student's grade is determined by his/her performance on the evaluation criteria and the grade assignments listed above.
<b>POLICY REGARDING INCOMPLETES:</b> Beginning specified date, a student that qualifies may request a grade of "I", incomplete. An "I" is a non-punitive grade given only if ALL three of the following criteria are satisfied. They are: 1) The student is passing the course; 2)The student has a justifiable (and verifiable) reason why the work cannot be completed as scheduled; and 3)The student arranges with the instructor to complete the work within one academic year.	
<b>FINAL GRADE:</b> Final grades online access: <a href="http://www.unt.edu/grades">http://www.unt.edu/grades</a>	
<b>DISABILITY ACCOMMODATIONS:</b> It is the responsibility of students with certified disabilities to provide the instructor with appropriate documentation from the Dean of Students Office.	

**Electronic access for homework assistance is available at:** [www.math.unt.edu/mathlab/emathlab](http://www.math.unt.edu/mathlab/emathlab)

Students are responsible for meeting all university deadlines (registration, fee payment, prerequisite verification, drop deadlines, etc.). See the printed Schedule of Classes and/or University Catalog for policies and dates.

## Course Content:

### Chapter R: Functions, Graphs, and Models

- R.1. Graphs and Equations
- R.2. Functions and Models
- R.3. Finding Domain and Range
- R.4. Slope and Linear Functions
- R.5. Nonlinear Functions and Models
- R.6. Mathematical Modeling and Curve Fitting

### Chapter 1: Differentiation

- 1.1. Limits: A Numerical and Graphical Approach
- 1.2. Algebraic Limits and Continuity
- 1.3. Average Rate of Change
- 1.4. Differentiation Using Limits of Difference Quotients
- 1.5. Differentiation Techniques: The Power and Sum-Difference Rules
- 1.6. Differentiation Techniques: The Product and Quotient Rules
- 1.7. The Chain Rule
- 1.8. Higher-Order Derivatives

### Chapter 2: Applications of Differentiation

- 2.1. Using First Derivatives
- 2.2. Using Second Derivatives
- 2.3\*. Graph Sketching: Asymptotes and Rational Functions
- 2.4. Using Derivatives to Find Absolute Maximum and Minimum Values
- 2.5. Maximum-Minimum Problems: Business and Economics Applications
- 2.6. Marginals and Differentials
- 2.7\*. Implicit Differentiation and Related Rates

### Chapter 3: Exponential and Logarithmic Functions

- 3.1. Exponential Functions
- 3.2. Logarithmic Functions
- 3.3. Applications: Uninhibited and Limited Growth Models
- 3.4\*. Applications: Decay
- 3.5. The Derivatives of  $a^x$  and  $\log_a x$
- 3.6. An Economics Application: Elasticity of Demand

### Chapter 4: Integration

- 4.1. The Area Under a Graph
- 4.2. Area, Antiderivatives, and Integrals
- 4.3. Area and Definite Integrals
- 4.4. Properties of Definite Integrals
- 4.5. Integration Techniques: Substitution
- 4.6. Integration Techniques: Integration by Parts

### Chapter 5: Applications of Integration

- 5.1: An Economics Application: Consumer Surplus and Producer Surplus
- 5.2: Applications of the Models  $\int_0^T P_0 e^{kt} dt$  and  $\int_0^T P_0 e^{-kt} dt$ .
- 5.4: Probability

\* Time permitting

## UNT Mathematics Core Component

After completing Math 1190, students will have learned:

1. to apply arithmetic, algebraic, geometric, higher-order thinking, and statistical methods to modeling and solving real-world situations;
2. to represent and evaluate basic mathematical information verbally, numerically, graphically, and symbolically;
3. to use appropriate technology to enhance mathematical thinking and understanding and to solve mathematical problems and judge the reasonableness of the results; and
4. to interpret mathematical models such as formulas, graphs, tables and schematics, and draw inferences from them.

While taking Math 1190, students will participate in the following over-arching objectives of UNT's core curriculum. Math 1190 students will:

- explore math
- make connections between different areas of knowledge and different ways of knowing
- be able to locate, evaluate and organize information including the use of information technologies
- think critically and creatively, learning to apply different systems of analysis
- develop problem solving skills
- cultivate self-responsibility, building a foundation for life-long learning