Issues in factorial design

No main effects but interaction present

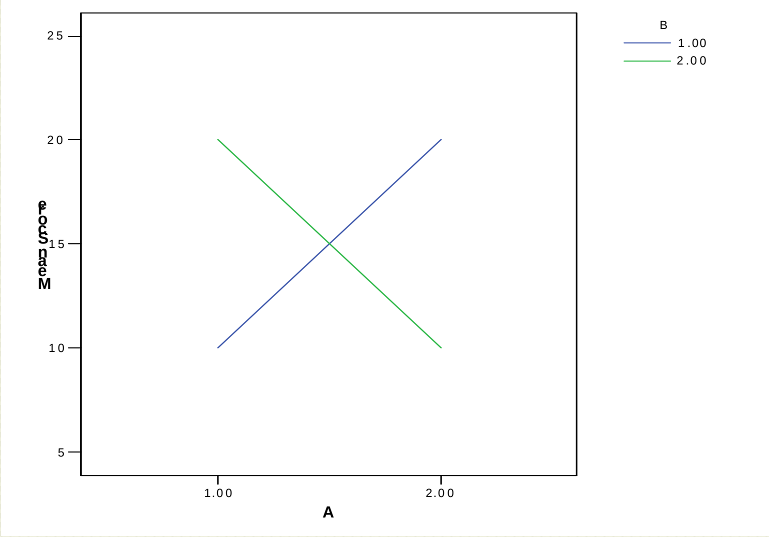
- Can I have a significant interaction without significant main effects?
- Yes
- Consider the following table of means

	B_1	B_2	
A_1	10	20	15
A_2	20	10	15
	15	15	

No main effects but interaction present

- We can see from the marginal means that there is no difference in the levels of A, nor difference in the levels of factor B
- However, look at the graphical display

	B_1	B_2	
A_1	10	20	15
A_2	20	10	15
	15	15	



No main effects but interaction present

- In such a scenario we may have a significant interaction without any significant main effects
- Again, the interaction is testing for differences among cell means after factoring out the main effects
- Interpret the interaction as normal

Unequal sample sizes

- Along with the typical assumptions of Anova, we are in effect assuming equal cell sizes as well
- In non-experimental situations, there will be unequal numbers of observations in each cell
  - Semester/time period for collection ends and you need to graduate
  - Quasi-experimental design
  - Participants fail to arrive for testing
  - Data are lost etc.
- In factorial designs, the solution to this problem is not simple
  - Factor and interaction effects are not independent
    - Do not total up to $SS_{b/t}$
  - Interpretation can be seriously compromised
  - No general, agreed upon solution

The problem (Howell example)

	Non-drinking							Drinking					Row means
Michigan	13	15	14	16	12			18	20	22	19	21	$\bar{Y}_{1**} = 18$
								23	17	18	22	20	
	$\bar{Y}_{11} = 14$							$\bar{Y}_{12} = 20$					
Arizona	13	15	18	14	10			24	25	17	16	18	$\bar{Y}_{2**} = 15.9$
	12	16	17	15	10	14							
	$\bar{Y}_{21} = 14$							$\bar{Y}_{22} = 20$					
Column means	$\bar{Y}_{*1*} = 14$							$\bar{Y}_{*2*} = 20$					

- Drinking participants made on average 6 more errors, regardless of whether they came from Michigan or Arizona
- No differences between Michigan and Arizona participants in that regard

Example

- However, there is a difference in the row means as if there were a difference between States
  - Michigan has worse drivers?
- This occurs because there are unequal number of participants in the cells
- In general, we do not wish sample sizes to influence how we interpret differences between means
- What can be done?

Another example

- How men and women differ in their reports of depression on the HADS (Hospital Anxiety and Depression Scale), and whether this difference depends on ethnicity.
- 2 grouping variables--Gender (Male/Female) and Ethnicity (White/Black/Other), and one dependent variable-- HADS score.

- Note the difference in gender
  - 2.47 vs. 4.73
- A simple t-test would show this difference to be statistically significant and noticeable effect

Report

HADS

Gender of subject	Ethnicity	Mean	N	Std. Deviation
Male	White	1.4800	133	1.6300
	Black	6.6000	10	1.7800
	Other	12.5600	9	2.7400
	Total	2.4729	152	3.3121
Female	White	2.7100	114	1.9600
	Black	6.2600	19	1.2400
	Other	11.9300	28	4.1100
	Total	4.7324	161	4.2419
Total	White	2.0477	247	1.8889
	Black	6.3772	29	1.4262
	Other	12.0832	37	3.7964
	Total	3.6351	313	3.9770

Unequal sample sizes

- Note that when the factorial anova is conducted, the gender difference disappears

- It's reflecting that there is no difference by simply using the cell means to calculate the means for each gender
  - (1.48+6.6+12.56)/3 vs. (2.71+6.26+11.93)/3

Tests of Between-Subjects Effects

Dependent Variable: HADS

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	3577.528[a]	5	715.506	161.854	.000
Intercept	5465.033	1	5465.033	1236.240	.000
SEX	.214	1	.214	.048	.826
ETHNICIT	2790.110	2	1395.055	315.574	.000
SEX * ETHNICIT	32.663	2	16.331	3.694	.026
Error	1357.151	307	4.421		
Total	9070.746	313			
Corrected Total	4934.680	312			

a. R Squared = .725 (Adjusted R Squared = .720)

1. Gender of subject

Dependent Variable: HADS

Gender of subject	Mean	Std. Error	95% Confidence Interval	
			Lower Bound	Upper Bound
Male	6.880	.328	6.235	7.525
Female	6.967	.218	6.537	7.396

Unequal sample sizes

- What do we do?
- One common method is the unweighted (i.e. equally weighted)-means solution
  - Average means without weighting them by the number of observations
    - Use the harmonic mean of our sample sizes
- Note that in such situations SS_{total} is usually not shown in reported ANOVA tables as the separate sums of squares effects do not usually sum to SS_{Model}

Unequal sample sizes

- In the drinking example, the unweighted means solution gives the desired result
  - No state difference
  - 17 v 17
- With the HADS data this was actually part of the problem
  - The t-test in isolation would be using the weighted means, the factorial anova the ‘unweighted’/equally weighted means
- However, with the HADS data the tests of simple effects would bear out the gender difference and as these would be part of the analysis, such a result would not be missed
- In fact the gender difference is largely only for the white category
  - i.e. there really was no *main effect* of gender in the anova design

A note about proportionality

- Unequal cells are not always a problem
- Consider the following tables of sample sizes

	B1	B2	B3
A1	5	10	20
A2	10	20	40

	B1	B2	B3
A1	5	20	10
A2	10	10	50

Proportionality

- The cell sizes in the first table are proportional b/c their relative values are constant across all rows (1:2:4) and columns (1:2)
- Table 2 is not proportional
  - Row 1 (1:4:2)
  - Row 2 (1:1:5)

	B1	B2	B3
A1	5	10	20
A2	10	20	40

	B1	B2	B3
A1	5	20	10
A2	10	10	50

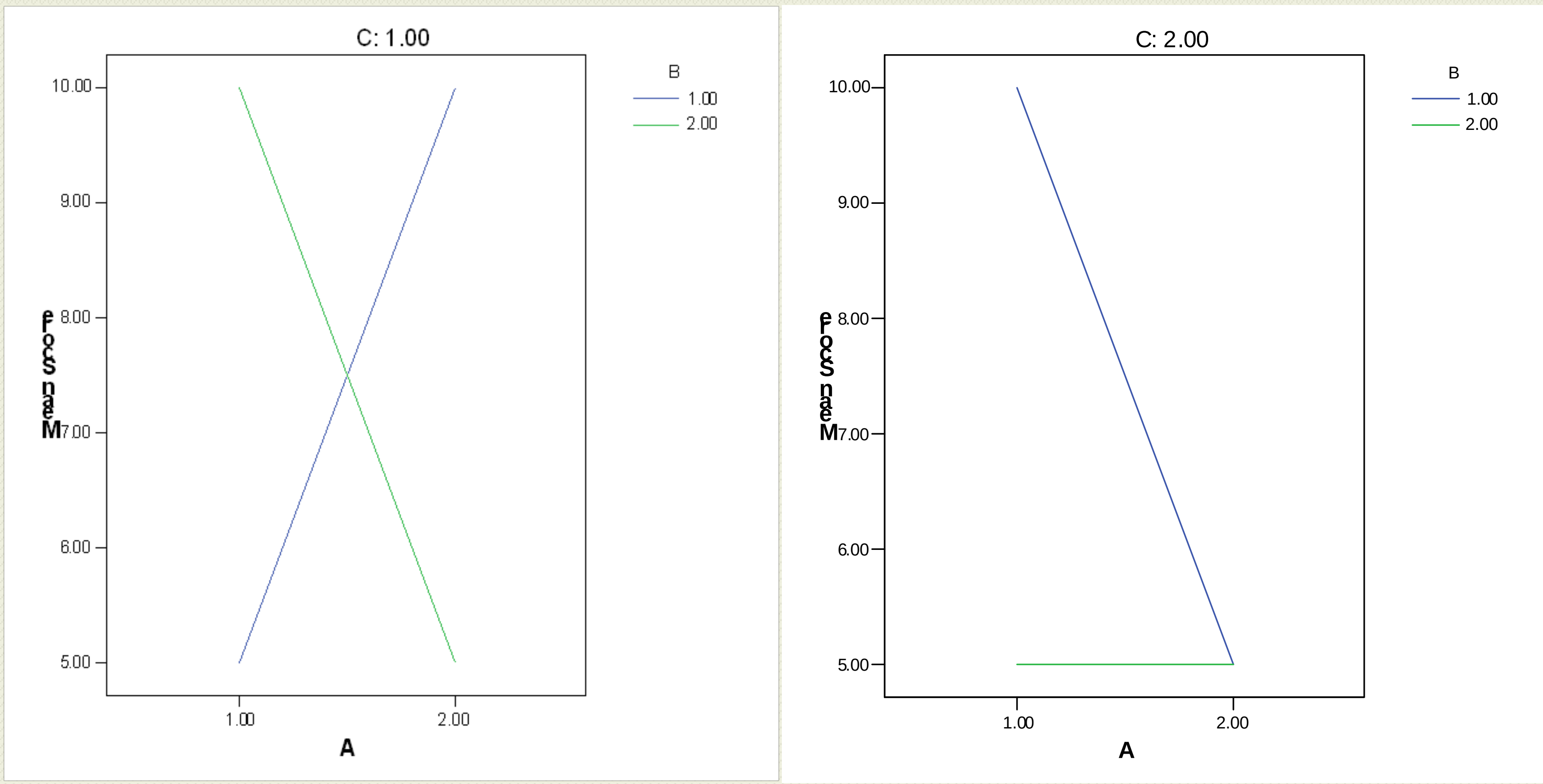
Proportionality

- Equal cells are a special case of proportional cell sizes
- As such, as long as we have proportional cell sizes we are ok with traditional analysis
- With nonproportional cell sizes, the factors become correlated and the greater the departure from proportional, the more overlap of main effects

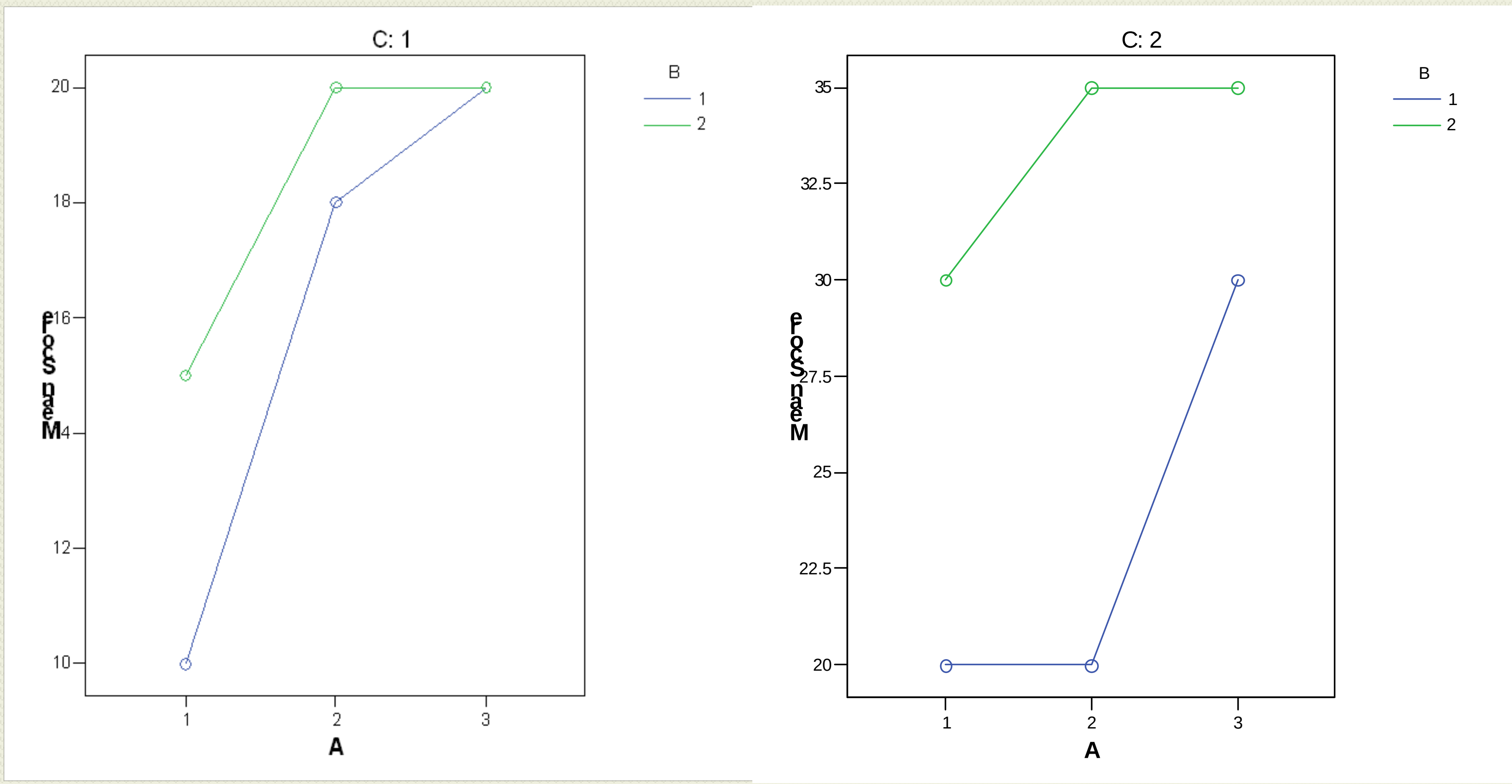
More complex design: the 3-way interaction

- Before we had the levels of one variable changing over the levels of another
- So what's going on with a 3-way interaction?
  - How would a 3-way interaction be interpreted?

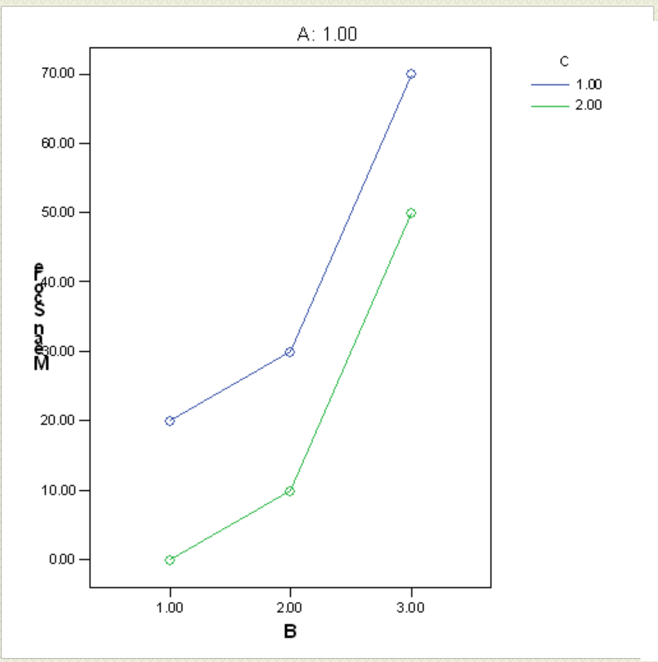
2 X 2 X 2 Example[1]

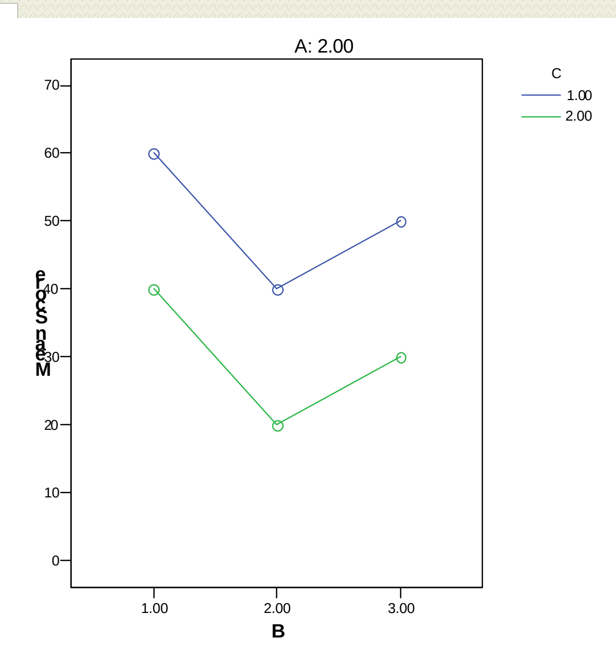


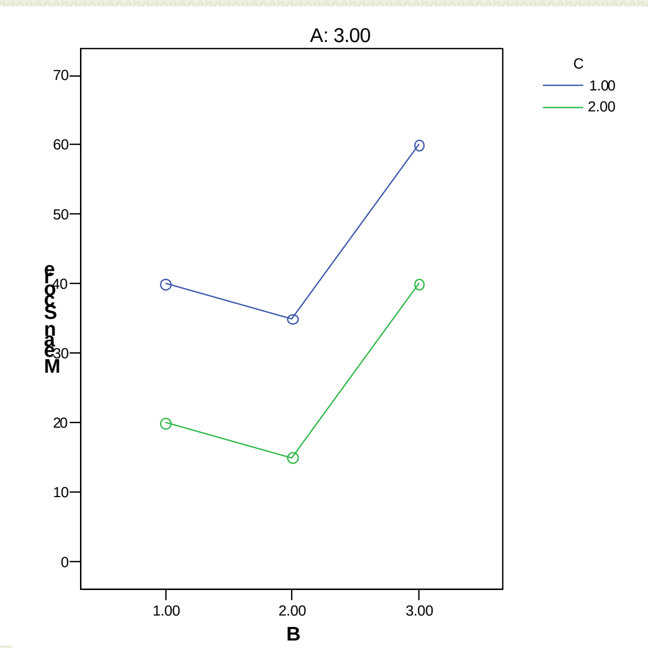
3 X 2 X 2



3 X 3 X 2


A: 1.00
C
1.00
2.00
70.00
60.00
50.00
40.00
30.00
20.00
10.00
0.00
1.00
2.00
3.00
B


A: 2.00
C
1.00
2.00
70
60
50
40
30
20
10
0
1.00
2.00
3.00
B


A: 3.00
C
1.00
2.00
70
60
50
40
30
20
10
0
1.00
2.00
3.00
B

Interpretation

- *An interaction between 2 variables is changing over the levels of another (third) variable*[1]
  - Interaction is interacting with another variable
  - A*B interaction depends on C
- Recall that our main effects would have their interpretation limited by a significant interaction
  - Main effects interpretation is not exactly clear without an understanding of the interaction
  - In other words, because of the significant interaction, the main effect we see for a factor would not be the same over the levels of another
- In a similar manner, our 2-way interactions' interpretation would be limited by a significant 3-way interaction

Simple effects

- Same for the 2-way interactions
- However now we have simple, simple main effects (differences in the levels of A at each BC) and simple interaction effects

Simple Main-Effects Sum of Squares

$$\sum_{k=1}^{q} SSA \text{ at } b_k = SSA + SSAB \qquad \sum_{l=1}^{r} SSB \text{ at } c_l = SSB + SSBC$$

$$\sum_{l=1}^{r} SSA \text{ at } c_l = SSA + SSAC \qquad \sum_{j=1}^{p} SSC \text{ at } a_j = SSC + SSAC$$

$$\sum_{j=1}^{p} SSB \text{ at } a_j = SSB + SSAB \qquad \sum_{k=1}^{q} SSC \text{ at } b_k = SSC + SSBC$$

Simple Simple Main-Effects Sum of Squares

$$\sum_{k=1}^{q}\sum_{l=1}^{r} SSA \text{ at } b_k c_l = SSA + SSAB + SSAC + SSABC$$

$$\sum_{j=1}^{p}\sum_{l=1}^{r} SSB \text{ at } a_j c_l = SSB + SSAB + SSBC + SSABC$$

$$\sum_{j=1}^{p}\sum_{k=1}^{q} SSC \text{ at } a_j b_k = SSC + SSAC + SSBC + SSABC$$

Simple Interaction-Effects Sum of Squares

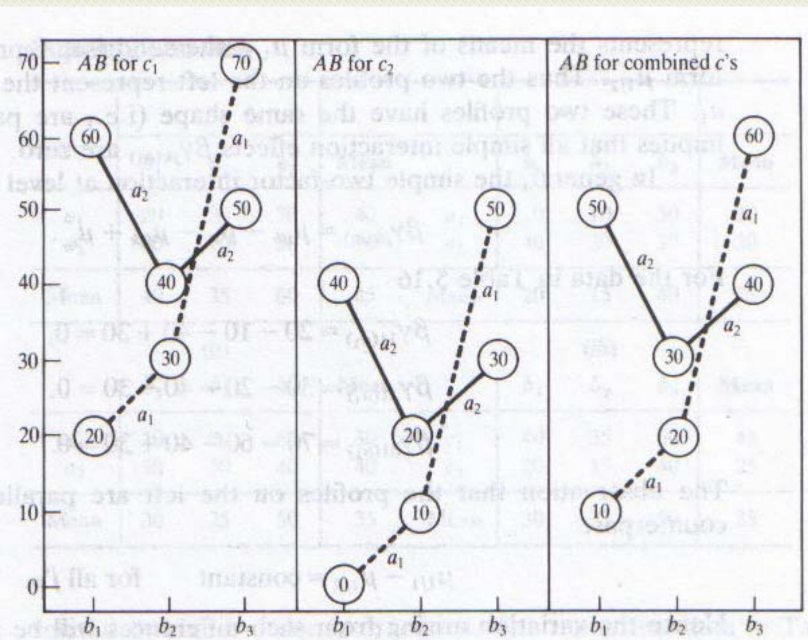
$$\sum_{l=1}^{r} SSAB \text{ at } c_l = SSAB + SSABC$$

$$\sum_{k=1}^{q} SSAC \text{ at } b_k = SSAC + SSABC$$

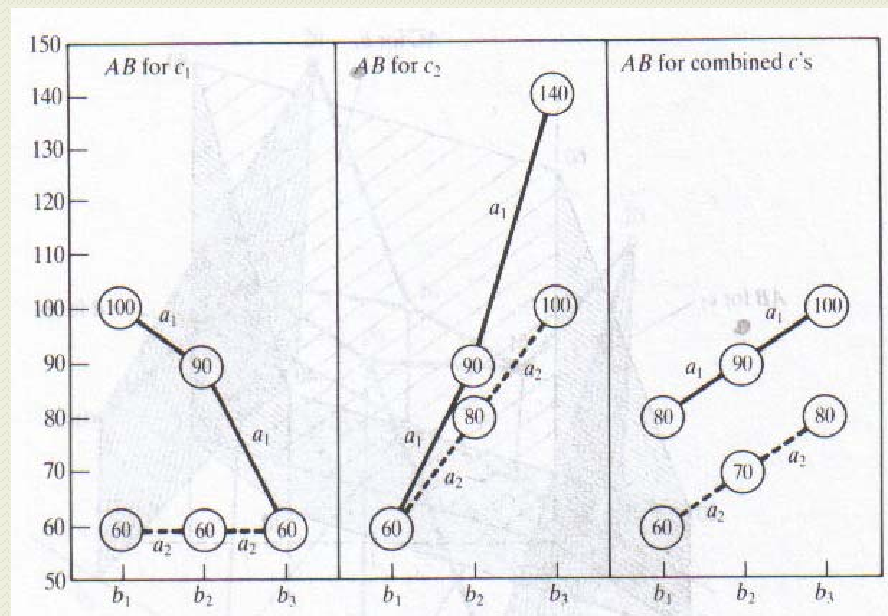
$$\sum_{j=1}^{p} SSBC \text{ at } a_j = SSBC + SSABC$$

Simple effects

- In this 3 X 3 X 2 example, the simple interaction of BC is nonsignificant, and that does not change over the levels of A (nonsig ABC interaction)
- Consider these other situations



$SS_{ab} \neq 0$, $SS_{abc} = 0$.



$SS_{ab} = 0$, $SS_{abc} \neq 0$.

Simple effects

- As mentioned previously, a nonsignificant interaction does not necessarily mean that the simple effects are not significant as simple effects are not just a breakdown of the interaction but the interaction plus main effect
  - In a 3-way design, one can technically test for simple interaction effects in the presence of a nonsignificant 3-way interaction
- The issue now arises that in testing simple, simple effects, one would have at minimum four comparisons (for a 2X2X2),
- Some examples from our backyard:
  - http://www.coe.unt.edu/brookshire/spss3way.htm