MORE MANOVA

Mancova
Factorial Manova and the
Path Analytic Perspective

Mancova

- Let's start with a brief overview of Ancova in the univariate case
- What is a covariate?
  - A continuous variable whose effects you want to control for (partial out) regarding the DV before looking at group differences on that DV
  - The covariate has a linear relationship with the DV

Mancova

- Testing for significantly different groups is conceptually the same as in Anova.
- We compare an estimate of the variance between groups to that which comes from within groups
- The difference is that we adjust the dependent variable by removing the variation that can be explained by the covariate
- What is left over is the adjusted DV, Y_{adj}

Mancova

- In Mancova the linear combination of DVs is adjusted for one or more covariates.
- The adjusted linear combinations of the DVs is the combination that would have been had all of the subjects scored the same on the covariate.
- For the significance tests, we'd go about them the same way as with regular Manova

Caveats to Mancova

- One should think hard before utilizing Ancova/Mancova
- 1. You need a theoretical reason for considering something to be a covariate, and want to have a minimum number of covariates
- 2. Many recommend against using covariates with intact groups (e.g. different schools)
  - This is because the adjustment still does not mean that the groups are equivalent on that covariate in reality
    - Essentially you are asking about what the data would look like if it weren't the data it is
  - Since we are dealing with intact groups, they may still differ on any number of characteristics which may cause unequal group means on **y** that are confounded with any treatment effects.
- 3. If one cannot imagine groups being equivalent on the covariate in the real world, then it doesn't make sense to use a procedure that equates them on that variable
  - Weight as a cv with gender an IV

Caveats to Mancova

- 4. Additional assumptions must be met for Ancova designs (homogeneity of regression)
- 5. The covariate should be independent of the IVs
  - E.g. the covariate cannot be measured after a treatment effect has been applied, as it would mean that part of what's being partialled out of the DV may actually be due to the treatment
- These all go with other issues/problems regarding Anova approaches in general
- Also note that other approaches may be available, particularly with pre post settings
  - Analyses on gain scores
  - Mixed Design
  - Solomon 4-group

Testing interactions

- A factorial MANOVA may be used to determine whether or not two or more categorical independent variables (and their interactions) significantly affect optimally weighted linear combinations of two or more normally distributed dependent variables.
- As in univariate factorial ANOVA, we will first take note of the interaction, as it will inform us whether to qualify any main effects or not

Interactions

- One could inspect the univariate interactions for each DV, but this does not really get at the heart of the matter with its disregard of DV correlations
- We could also, in a similar fashion as we suggested in the one way design as a post hoc, perform multivariate one-way designs for each level of the second variable

Interactions

- However there is a problem this time around with such an approach with interactions
- With the interaction setting, neither set of weights would likely be the same as that which maximized the multivariate interaction

Interactions

- A significant multivariate interaction means that the effect of one IV depends upon the levels of another IV, where the multivariate DV is one or more canonical variates
- Compute each subjects' canonical variate scores for each root that is significant and then do simple effects analysis on the corresponding canonical variate.
  - i.e. Univariate anovas on the variate scores
  - Note this would be an option in the one-way MANOVA scenario as well as we have discussed

Example

- 2 X 4 Manova
- Achievement (high vs. low)
- Teaching method (4 kinds)
- 2 tests (social studies and science)

Achieve	Method	Test1	Test2
1	1	6	10
1	1	7	8
1	1	9	9
2	1	11	8
2	1	7	6
2	1	10	5
1	2	13	16
1	2	11	15
1	2	17	18
2	2	10	12
2	2	11	13
2	2	14	10
1	3	9	11
1	3	8	8
1	3	14	9
2	3	4	12
2	3	10	8
2	3	11	13
1	4	21	19
1	4	18	15
1	4	16	13
2	4	11	10
2	4	9	8
2	4	8	15

Output

- Significant main effects and interaction

Multivariate Tests[c]

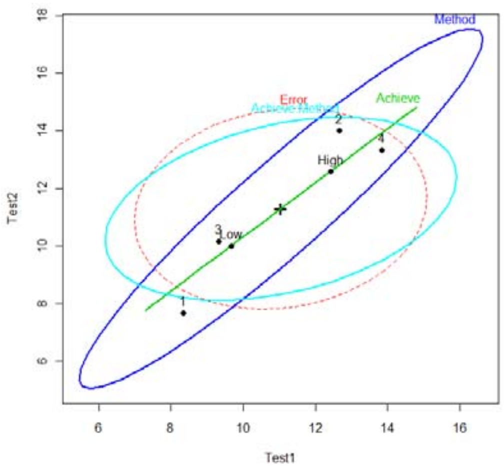
Effect		Value	F	Hypothesis df	Error df	Sig.	Partial Eta Squared
achieve	Pillai's Trace	.454	6.238	2.000	15.000	.011	.454
	Wilks' Lambda	.546	6.238	2.000	15.000	.011	.454
	Hotelling's Trace	.832	6.238	2.000	15.000	.011	.454
	Roy's Largest Root	.832	6.238	2.000	15.000	.011	.454
teachmethod	Pillai's Trace	.817	3.682	6.000	32.000	.007	.408
	Wilks' Lambda	.245	5.097	6.000	30.000	.001	.505
	Hotelling's Trace	2.825	6.593	6.000	28.000	.000	.586
	Roy's Largest Root	2.733	14.576[b]	3.000	16.000	.000	.732
achieve * teachmethod	Pillai's Trace	.779	3.406	6.000	32.000	.010	.390
	Wilks' Lambda	.365	3.280	6.000	30.000	.013	.396
	Hotelling's Trace	1.347	3.143	6.000	28.000	.017	.402
	Roy's Largest Root	.916	4.883[b]	3.000	16.000	.013	.478

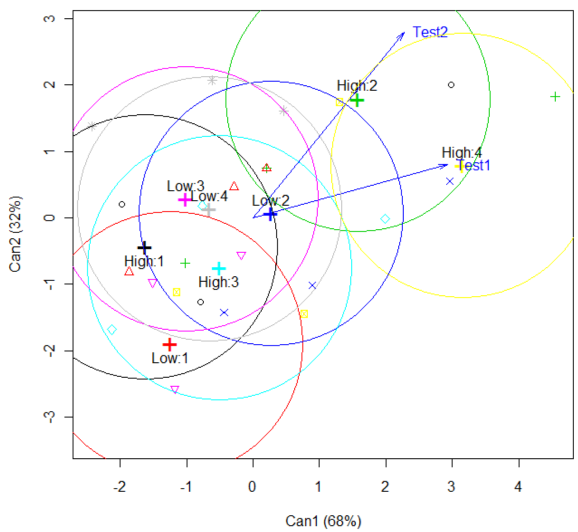
b. The statistic is an upper bound on F that yields a lower bound on the significance level.

c. Design: Intercept+achieve+teachmethod+achieve * teachmethod

Interaction interpretation

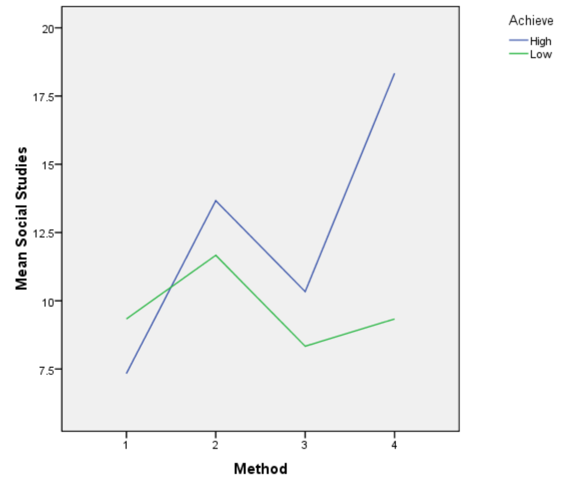
- Here we can see that all effects are statistically significant
  - There ellipse extends beyond the error ellipse
- While the main effect group differences lie along both DVs, the interaction variance lies more along Test1 (though not to the relative exclusion of Test 2)
- In terms of each of the cells, we can see High Achiever Methods 2 and 4 (green and yellow circles) are distinct from the others, while Low Achiever Method 1 is notably lower on the canonical variates relative to other groups

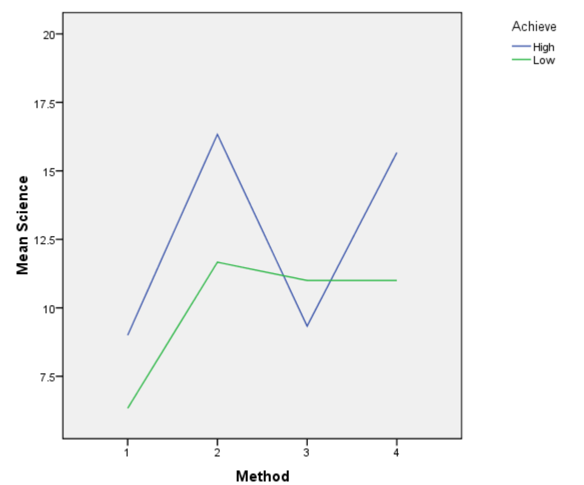




Interaction interpretation

- We can see this reflected to some extent in the inspection of univariate graphs graphs
  - Soc (top) and Sci (bottom)
- For the Soc test, methods 2 and 4 work best for high achiever, while there is little distinction for low achievers
  - High achievers do better than low with every method but 1
- For science we see a similar pattern for though smaller range of scores for high achievers (methods 2 and 4 are now indistinguishable), while for low it appears method 1 is worse, but there is again not much distinction among the rest
  - High achievers do better for every method but 3
- Significantly so?





Interaction interpretation

- Most packages do not seem to allow for a easy test of multivariate simple main effects for the interaction
  - In fact, the topic is pretty much ignored in most texts, or only discussed for univariate followups
- A conceptually simple approach that would keep the mulitvariate spirit would involve testing the simple effects on the canonical variate scores
- Look for differences of method at levels of high or low achievement or vice versa.

Univariate Tests

Dependent Variable: Dim1

achieve		Sum of Squares	df	Mean Square	F	Sig.	Partial Eta Squared
1.00	Contrast	41.148	3	13.716	13.687	.000	.720
	Error	16.034	16	1.002			
2.00	Contrast	3.997	3	1.332	1.330	.300	.200
	Error	16.034	16	1.002			

Each F tests the simple effects of teachmethod within each level combination of the other effects shown. These tests are based on the linearly independent pairwise comparisons among the estimated marginal means.

High Achievers

Low Achievers

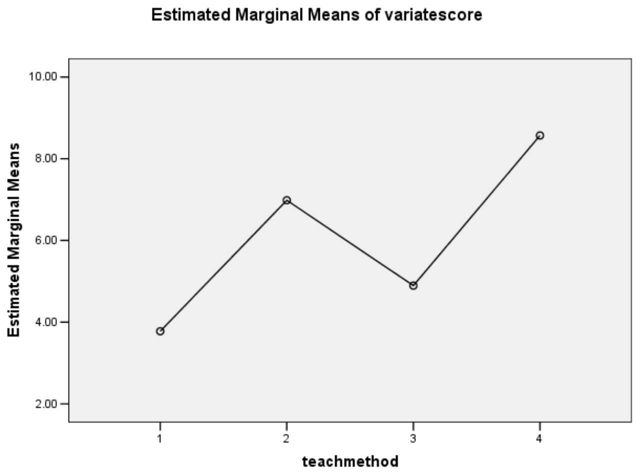
Simple Effects

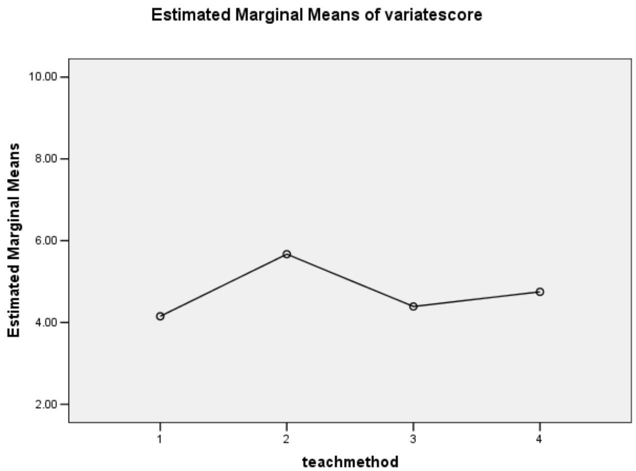
- Recall that simple effects are not just a breakdown of an interaction, but the variance attributable to the interaction and the main effect being studied
  - If A at levels of B the variance in the simple effects come from variance of A and A*B
- While your interaction tells you the simple effects are different, the simple effects tell are telling you just whether the mean differences for that grouping variable are significantly different from zero (just like a one-way ANOVA)*

Graphically

- Sig difference of method for high achievers but not for low achievers
- One could then perform more post hocs or go by a descriptive account

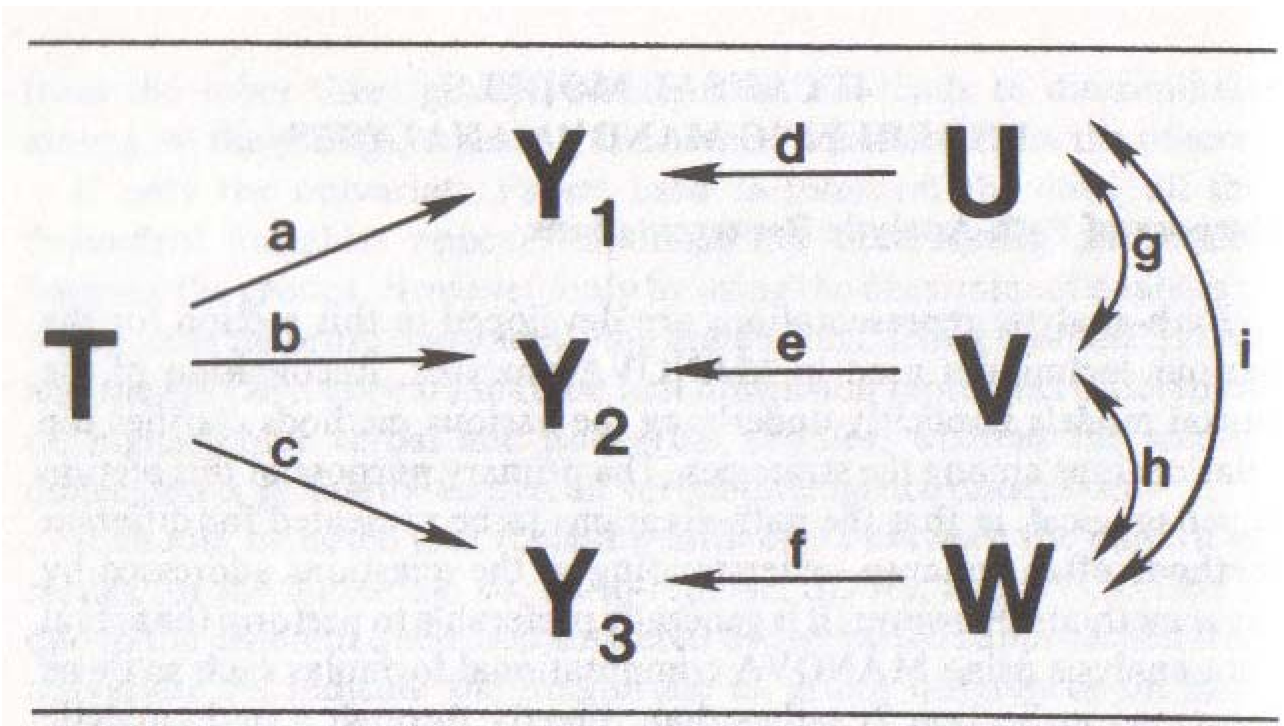
- Methods 4 seems to be best and 1 worst for high achievers but low achievers aren't really showing any differences



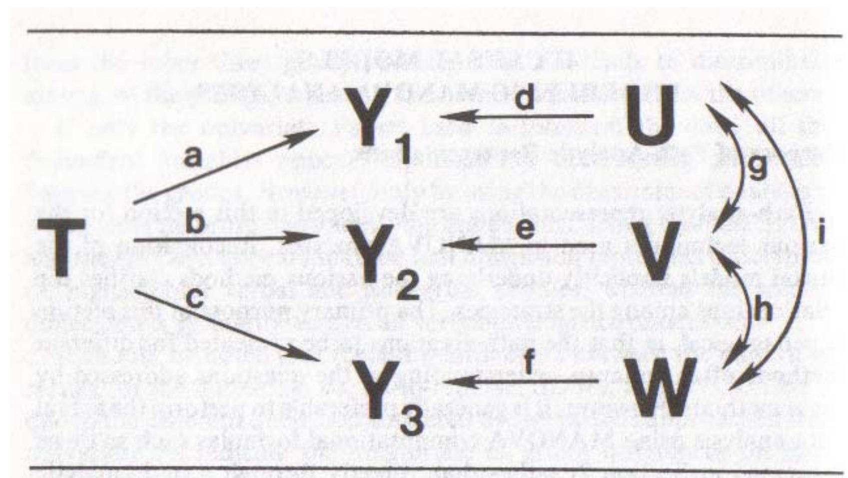


Causality in Manova

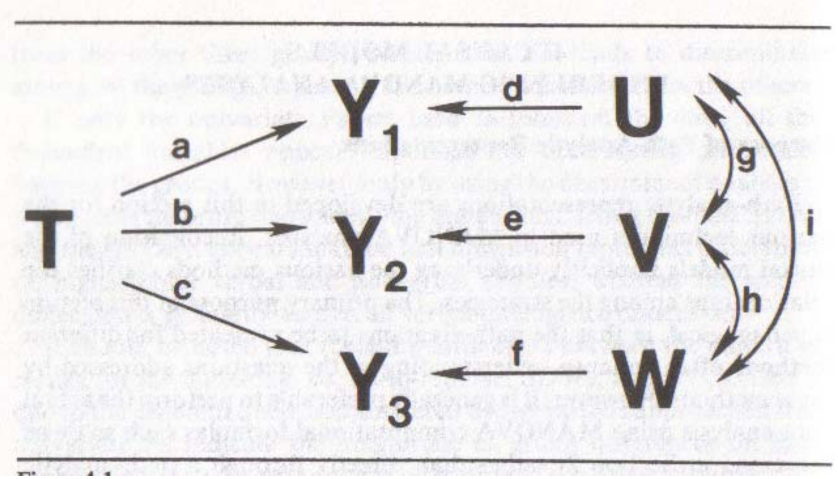
- Path analytic approach to Manova



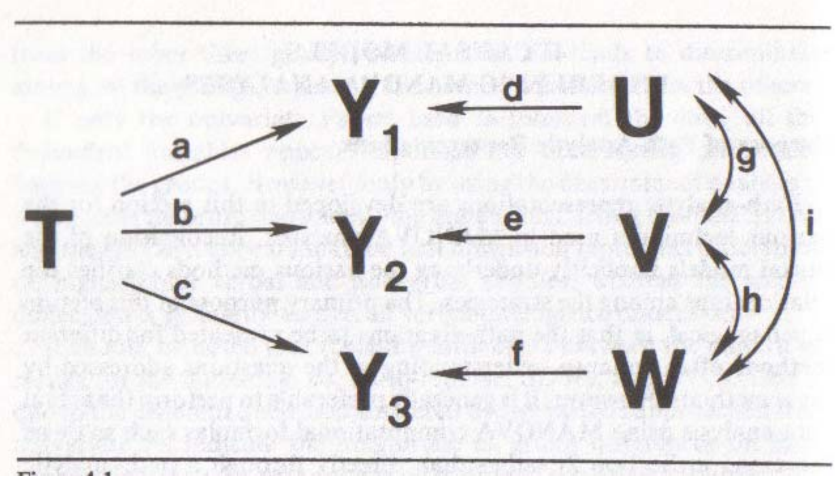
- Consider a 2 group 3 DV set up as here
- T the dummy coded grouping variable
- Ys are the DVs
- Arrows are paths
- Lower case letters coefficients representing the strength of that path



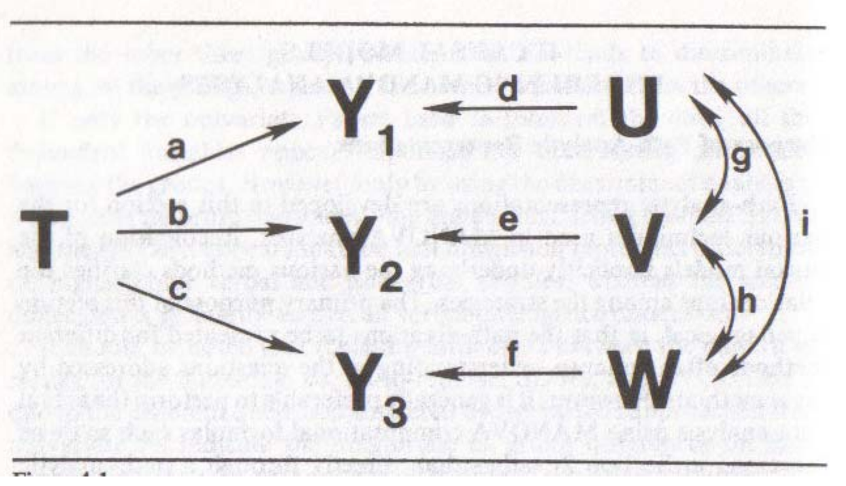
- U, V, and W are 'disturbance' terms i.e. error
  - Source of variability in the DVs not accounted for by the model
  - Example, U represents all causes of Y_1 not caused by the treatment assignment
- The disturbances of the DVs are correlated with one another, though precisely how (causally) is not specified
  - Hence double arrows



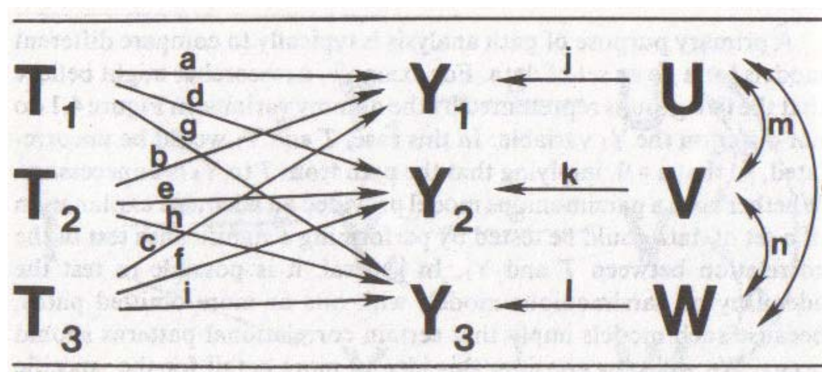
- The omnibus Manova null hype is that the treatment group means are the same on the DVs
- In terms of the path analysis, we are saying the treatment has zero correlation with the DVs
  - Coefficients = 0
- The test can be seen as a comparison of a (full) model such as the figure vs. one (reduced) in which there are no causal paths extending from T



- A follow up of univariate tests can be seen here as well
- The omnibus rejected the idea that the coefficients = 0
- The univariate approach would be testing whether each particular coefficient = 0

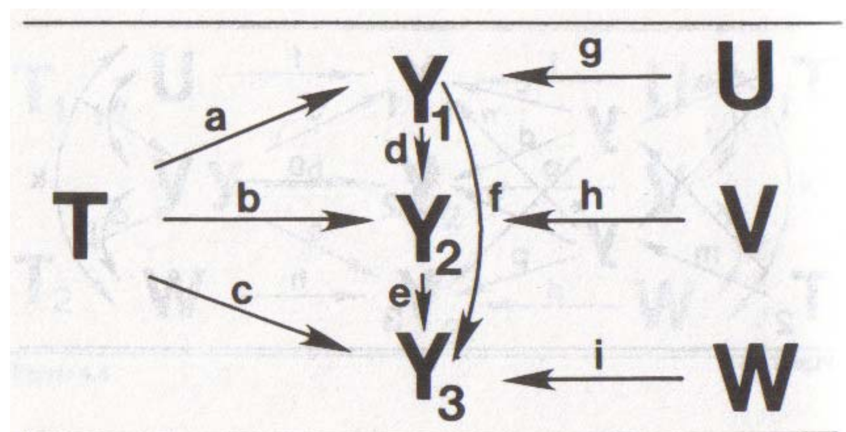


- Here is a case involving 4 treatment groups
- K-1 = 3 dummy variables
- The same approach applies, i.e. the omnibus test of whether any coefficients a-i are significantly different from zero
- A simultaneous test of a,b, and c would be equivalent to the unvariate test on Y_1



Step-down test

- This diagram represents a two group IV with 3 DVs
- The equations are indicative of the regression approach, but the analysis can also be thought of as an Ancova with any prior DVs as the covariate(s)
- Note that unlike previously, this suggests a causal ordering to the DVs rather than a simple correlation



$$Y_1 = aT + gU$$

$$Y_2 = dY_1 + bT + hV$$

$$Y_3 = fY_1 + eY_2 + cT + iW$$